ON THE SOLUTION OF SOME PROBLEMS OF THE THEORY OF PROBE MEASUREMENTS OF SEMICONDUCTOR FILM PARAMETERS

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Many problems of the theory of probe measurements of semiconductor film parameters reduce Neumann value problems [1 to 3] for the Laplace equation with inhomogeneous boundary conditions. Their solution is of great value in connection with the problem of microminiaturization of radio electronics equipment.

Boundary value problems with separable variables and inhomogeneous conditions most frequently reduce to problems with homogeneous boundary conditions, which are then usually solved by the Fourier method [4]. To do this, the solution is represented as the sum of functions, each of which is subject to the original equation and homogeneous conditions. Such a method turns out to be inapplicable for the majority of Neumann problems since the Ostrogradskii-Gauss theorem [5] is not satisfied separately for the new desired functions. However, this latter difficulty may be eliminated if the new functions are subject to another more general equation rather than the original one.

As an example let us consider a Neumann problem of the Laplace equation in the form

$$\nabla^2\varphi(x, y) = 0, \qquad \varphi_x = p_{1,2}(y) \quad \text{for} \quad x = \mp a, \qquad \varphi_y = q_{1,2}(x) \quad \text{for} \quad y = \mp b \qquad (1)$$

Let us seek the solution as

$$\varphi(x, y) = u(x, y) + v(x, y) \qquad (2)$$

and let us require that the functions u and v satisfy equations and boundary conditions

$$\nabla^2 u(x, y) = \rho, \qquad u_x = p_{1,2}(y) \quad \text{for} \quad x = \mp a, \qquad u_y = 0 \quad \text{for} \quad y = \mp b \qquad (3)$$

$$\nabla^2 v(x, y) = -\rho, \qquad v_x = 0 \quad \text{for} \quad x = \pm a, \qquad v_y = q_{1,2}(x) \quad \text{for} \quad y = \mp b \qquad (4)$$

The original equation (1) will be satisfied for any choice of the function $\rho(x, y)$. In order to simplify the equations for u and v as much as possible, let us consider ρ a constant, and let us select its value so that the Ostrogradskii-Gauss theorem will be satisfied for the functions u and v, i.e. let us put

$$\rho = \frac{1}{4ab}\int_{-b}^{b}[p_2(y) - p_1(y)]\,dy = -\frac{1}{4ab}\int_{-a}^{a}[q_2(x) - q_1(x)]\,dx \qquad (5)$$

In this case the problems for u and v will be solvable and they may be integrated by the Fourier method. If we put $\rho = 0$, i.e. subject u and v to the original Laplace equation, it is then seen from (5) that the Ostrogradskii-Gauss theorem will not be satisfied in the general case.

From the viewpoint of field theory, the ρ in (3) plays the part of the electrical charge density. Hence, the transition to these equations means the simultaneous introduction of two equal charges with opposite sign and constant density. The magnitude of the latter is selected so that the fluxes of the fields they create would equal the fluxes of the field in conformity with the boundary conditions of the problem.

Let us take the particular case presented in [5] as an example of the problem not solvable by separation of variables, and when $p_1 = q_1 = 0$, $p_2 = I/2b\sigma$ and $q_2 = -I/2a\sigma$, Where I is the electric current intensity, σ is the conductivity of the sample.

In this case Equations (3) become

$$\nabla^2 u(x, y) = {}^1\!/_4 I / ab\sigma, \quad u_x(-a, y) = 0, \quad u_x(a, y) = {}^1\!/_2 I / b\sigma, \quad u_y(x, \mp b) = 0 \quad (6)$$

$$\nabla^2 v(x, y) = -{}^1\!/_4 I / ab\sigma, \quad v_x(\mp a, y) = 0, \quad v_y(x, -b) = 0, \quad v_y(x, b) = -{}^1\!/_2 I / a\sigma$$

Their integration by the Fourier method yields at once

$$u = \frac{I}{8ab\sigma}(x + a)^2, \qquad v = -\frac{I}{8ab\sigma}(y + b)^2 + c \quad (7)$$

Solution of the problem by using the Green's function leads to an expression for φ in the form of two Fourier series, and only after they are summed, we obtain terms (7). If the problem is solved by the Fourier method without its being reduced to a problem with homogeneous boundary conditions [5], then we find one of the terms (7) also just in the form of a Fourier series. Hence, the method presented here of solving the problem turns out to be simplest and briefest.

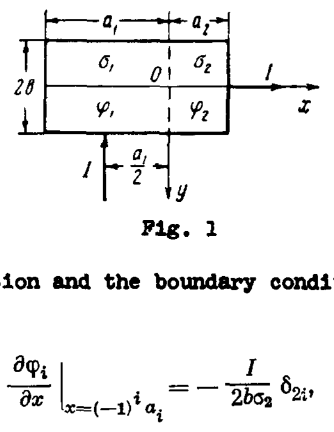

Fig. 1

The proposed method of solving the Neumann problem with inhomogeneous boundary conditions also has considerable generality. Let us consider an example with singularities, characteristi for a number of problems of the theory of probe measurements of semiconductor film parameters. In the rectangular sample shown in Fig.1 and consisting of two portions with conductivities σ_1 and σ_2, a current I is passed through one edge by using a point probe, and is drained uniformly off the other. It is necessary to find the potential distribution of the field which satisfies the Laplace equation and the boundary conditions

$$\nabla^2 \varphi_i(x, y) = 0 \quad (8)$$

$$\frac{\partial \varphi_i}{\partial x}\bigg|_{x=(-1)^i a_i} = -\frac{I}{2b\sigma_2}\delta_{2i}, \qquad \frac{\partial \varphi_i}{\partial y}\bigg|_{y=-b} = 0, \qquad \frac{\partial \varphi_i}{\partial y}\bigg|_{y=b} = \frac{I}{\sigma_1}\delta\left(x + \frac{1}{2}a_1\right)\delta_{1i}$$

$$(\varphi_1 - \varphi_2)_{x=0} = 0, \qquad \left(\sigma_1 \frac{\partial \varphi_1}{\partial x} - \sigma_2 \frac{\partial \varphi_2}{\partial x}\right)_{x=0} = 0$$

in each portion of the sample, where δ_{ij} is the Kronecker delta, and $\delta(x)$ is the delta-function. It is not possible to solve this problem by using the Green's function or the Fourier method directly since explicit knowledge of $\partial\varphi_i/\partial n$ at all points of the domain surfaces of the sample is required in these methods. But only the relationship between $\partial\varphi_1/\partial x$ and $\partial\varphi_2/\partial x$ is given on the boundary $x = 0$, and they are themselves unknown.

Equation (8) is integrated without any difficulty by the method expounded above. Let us put

$$\varphi_1 = u_1(x, y) + v_1(x, y), \qquad \nabla^2 u_1 = -\nabla^2 v_1 = -{}^1\!/_2 I / a_1 b\sigma_1 \quad (9)$$

and let us require that the functions u_1 and v_1 satisfy the boundary conditions

$$\left.\frac{\partial v_1}{\partial x}\right|_{x=\mp a} = 0, \qquad \left.\frac{\partial v_1}{\partial y}\right|_{y=-b} = 0, \qquad \left.\frac{\partial v_1}{\partial y}\right|_{y=b} = \frac{I}{\sigma_1}\,\delta\left(x + \frac{1}{2}\,a_1\right) \tag{10}$$

$$\left.\frac{\partial u_1}{\partial x}\right|_{x=-a_1} = 0, \qquad (v_1 + u_1 - \varphi_2)_{x=0} = 0, \qquad \left(\sigma_1\frac{\partial u_1}{\partial x} - \sigma_2\frac{\partial \varphi_2}{\partial x}\right)_{x=0} = 0, \qquad \left.\frac{\partial u_1}{\partial y}\right|_{y=\mp b} = 0$$

The obtained problems for v_1, u_1 and φ_2 may now be solved by the Fourier method, first to determine v_1 and then to find u_1 and φ_2. As a result we obtain for the field potential

$$\varphi_i = \frac{I}{4a_1b\sigma_1}\left[(y+b)^2 - (x+a_1)^2 + 8b\sum_{k=2,4\ldots}(-1)^{k/2}\frac{\cosh\alpha_k(y+b)}{\alpha_k\sinh 2\alpha_k b}\cos\alpha_k x\right]\delta_{1i} -$$
$$- \frac{I}{b\sigma_2}\,x\delta_{2i} + \frac{I\sigma_{i-(-1)^i}}{b\sigma_1}\sum_{n=1,2\ldots}(-1)^n\frac{\cosh\alpha_n\left[x + (-1)^{i+1}a_i\right]}{\alpha_n D_n \sinh{}^1\!/_2\alpha_n a_1 \sinh\alpha_n a_i}\cos\alpha_n(y+b) + c_1\delta_{2i} + c \tag{11}$$

where the α_k, α_n, c_1 and D_n are defined by Equations

$$\alpha_k = \frac{\pi k}{a_1}, \quad \alpha_n = \frac{\pi n}{2b}, \quad c_1 = \frac{I}{b\sigma_1}\left(\frac{4}{3}\,\frac{b^2}{a_1} - \frac{9}{32}\,a_1\right)$$
$$D_n = \sigma_1\coth\alpha_n a_2 + \sigma_2\coth\alpha_n a_1 \tag{12}$$

The expounded method is applicable to solve the Neumann problem of the Poisson equation with inhomogeneous boundary conditions and other problems.

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